Trigonometric Solution for Angled Box Side Rotation
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The question is as follows: given that the sides of a square box have been cut at an angle of 71.3° with respect to the horizion, what angle ϕ do you rotate the sides so that the upper corners of the four sides are perfectly joined. While it is possible to have SketchUp construct a graphical solution, the solution presented here will use only trigonometry.

The box geometry before side rotation is shown below in Figure 0.1:

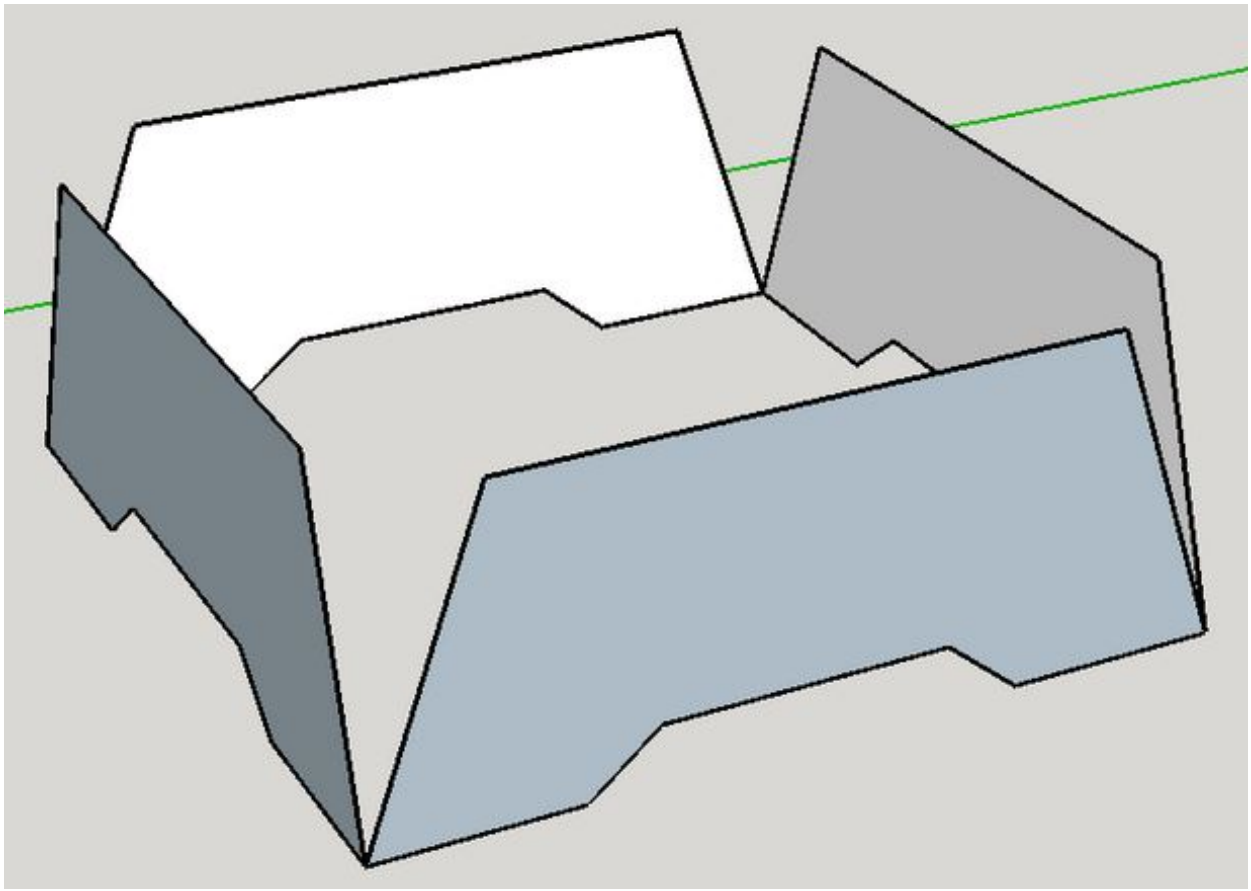

Figure 0.1: Box with angled sides before rotation into final position.

The trigonometry solution for the problem is

$$\cos\phi = \frac{1}{\tan\theta}; \text{for } \theta = 71.3^\circ,\ \phi = 70.21562... \tag{0.1}$$

where θ is given for the problem. The discussion that follows will present the derivation of this result.

Using SketchUp, a sketch was created to define some nomenclature; this result is shown in Figure 0.2.

The angle ϕ through which the sides must be rotated through is defined in Figure 0.3. The geometry in Figure 0.3 tells us that

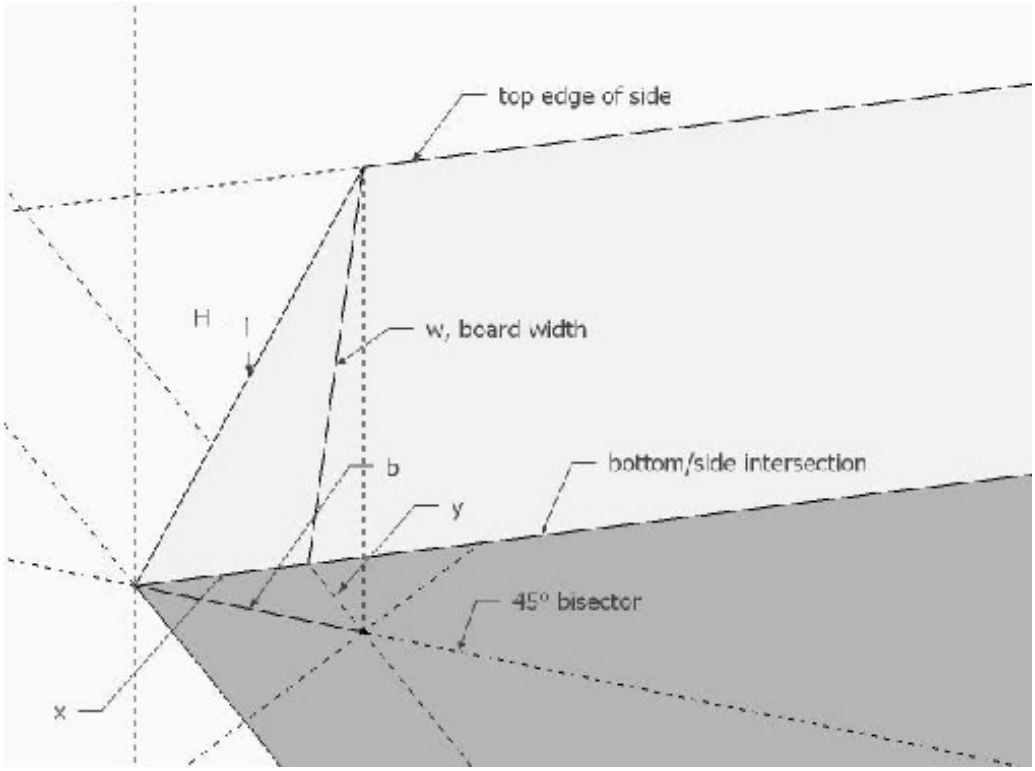

Figure 0.2: 3-D sketch of intersection between rotated side and base of box.

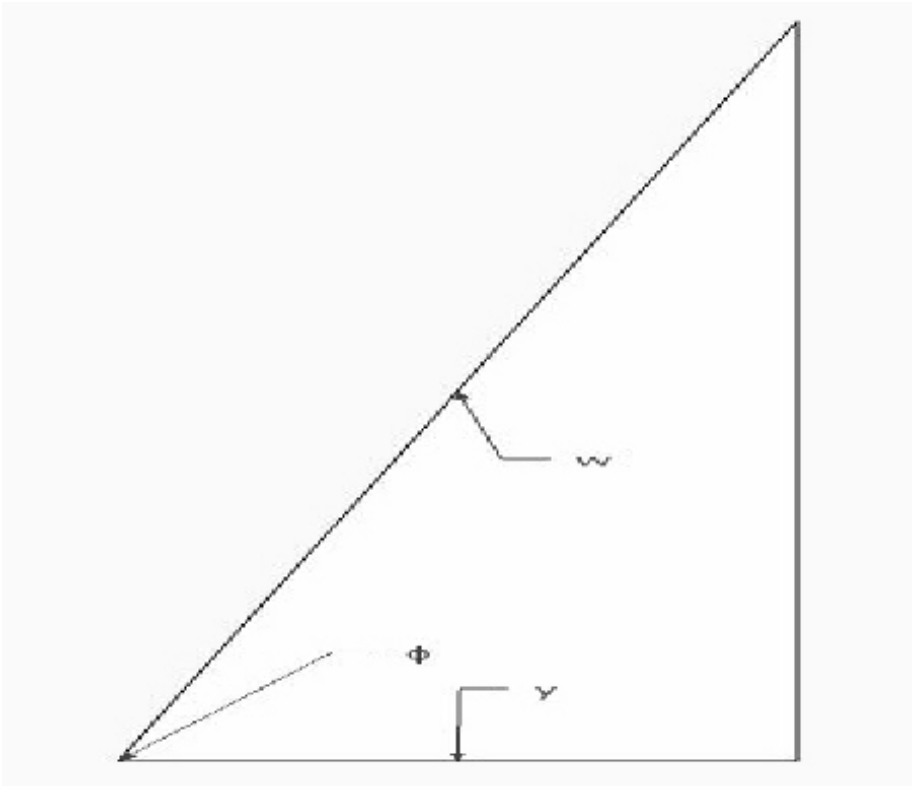

Figure 0.3: Definition of angle ϕ through which side must be rotated.

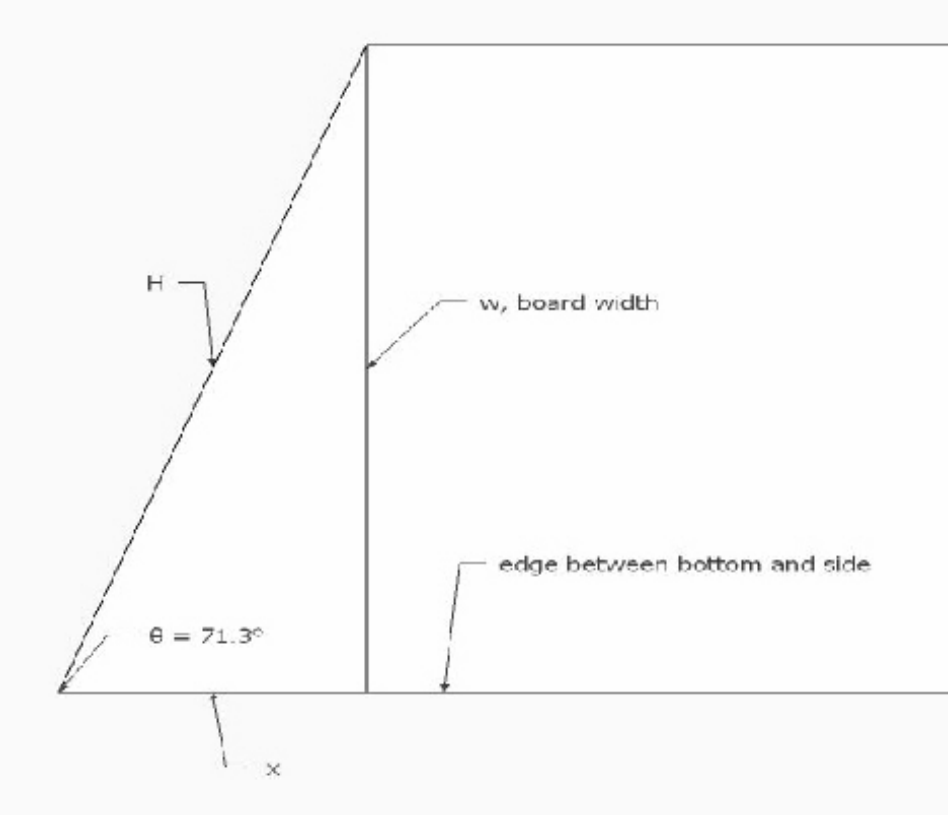

Figure 0.4: Sketch of box side with angled cut of 71.3°.

$$\cos\phi = y/w \tag{0.2}$$

Since the length b in Figure 0.2 lies along the 45° bisector, the sides x and y are of identical length.

The remaining step is to relate the side length x (or y) to the board width w using Figure 0.4.

$$\tan\theta = w/x = w/y \tag{0.3}$$

Combining Eqs. (0.2) and (0.3) yields the final result

$$\cos\phi = \frac{1}{\tan\theta} \tag{0.4}$$